

A, B fixed

C: free but not moving

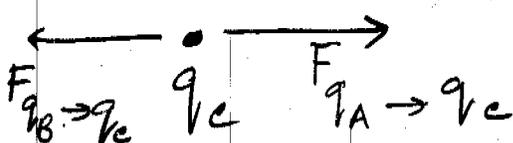
(equilibrium)

Equilibrium implies forces on q_C are balanced. Since q_A & q_B are on the same side of q_C , q_A & q_B must have the opposite signed charge to exert oppositely directed forces on q_C

Assuming q_A is same sign as q_C

q_B is opposite sign as q_C

then



Forces must be equal.

Quick: notice q_A is twice as far to q_C as q_B to q_C . So q_A must be $2^2 = 4$ times larger to exert the same force since $F \propto \frac{qQ}{r^2}$. $\therefore \boxed{q_A = -4q_B}$ minus because opposite sign.

Not so quick: $|F_{q_A \rightarrow q_C}| = k \frac{q_A q_C}{(2d)^2} = k \frac{q_A q_C}{4d^2}$

$$|F_{q_B \rightarrow q_C}| = k \frac{q_B q_C}{d^2}$$

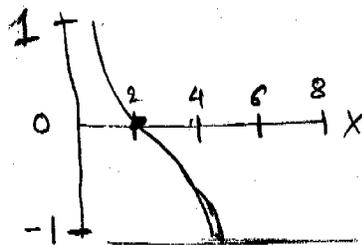
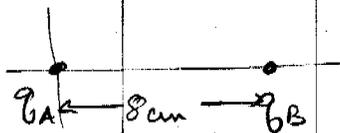
Since sizes of force are equal

$$k \frac{q_A q_C}{4d^2} = k \frac{q_B q_C}{d^2} \quad \text{or} \quad \underline{q_A = 4q_B}$$

of course taking signs of charges into account

$$\underline{q_A = -4q_B}$$

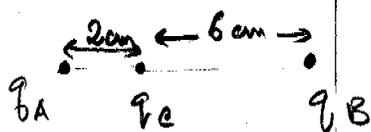
Ch 22
P 99



From the graph: (i) $F_{c, net} = 0 @ x = 2 \text{ cm}$
 \rightarrow closer to q_A . So q_B must be larger than q_A

Also since $F_{c, net} = 0$ at one point in between A & B, then

q_A & q_B must have the same signed charge!

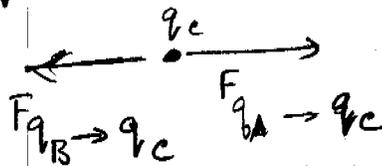


Since forces are balanced when q_C is 3 times as far from q_B as from q_A , $q_B = 3^2 q_A = 9 q_A$

(Since in Coulombs Law $F \propto \frac{q}{r^2}$)

We still don't know if q_A is positively charged or negatively charged.

$q_C = +5 \text{ e}$. Suppose q_A was +. Then $q_C = +$ also
 then forces on q_C



If $x = 2$ $F_{q_B \to q_C} = F_{q_A \to q_C}$

for $x < 2$, $F_{q_A \to q_C} > F_{q_B \to q_C} \Rightarrow F_{c, net} > 0$

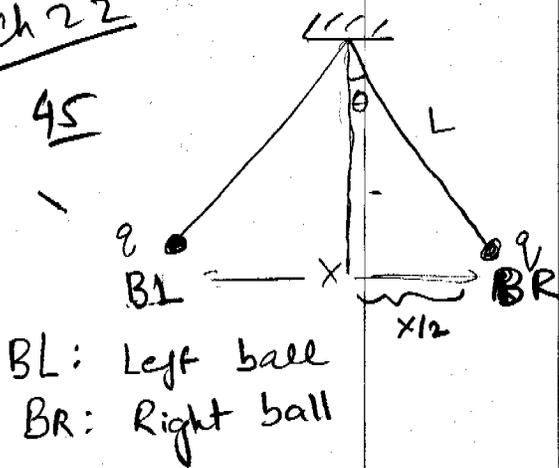
Awesome! See that matches the graph!
 $F_{c, net} > 0$ for $x < 2 \text{ cm}$.

That confirms that q_A is positively charged

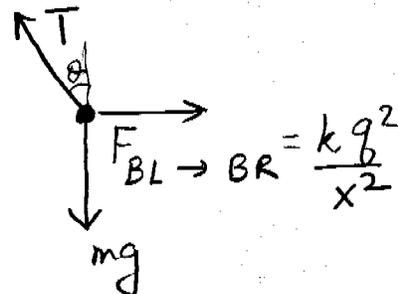
& $q_B = 9 q_A$

Ch 22

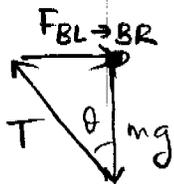
45



Forces on Right ball



The sum of these forces is 0. Since Ball is in equilibrium.



I can add the 3 vectors like this.

Since they complete the triangle

$$F_{\text{net}} = 0 \Rightarrow \tan \theta = \frac{F_{BL \to BR}}{mg} = \frac{kq^2}{x^2 mg}$$

From the geometry of the hanging balls

$$\sin \theta = \frac{x/2}{L}$$

& by the assumption in the question

$$\tan \theta = \sin \theta$$

$$\Rightarrow \frac{kq^2}{x^2 mg} = \frac{x/2}{L} = \frac{x}{2L}$$

$$\text{or, } x = \left(\frac{2kq^2 L}{mg} \right)^{1/3}$$

You can do the numerical part yourself, I trust!

⊗ You could also do the forces by breaking T into components $T \cos \theta$ & $T \sin \theta$

CA22
P56

Well (c) & (d) are identical. (Printing it again doesn't change the force)

↳ Seriously, I think it's a typo.

(b) is just (c) rotated. So the size of force should be same for (b) & (c)

So For magnitude of force

$$(b) = (c) = (d)$$

For (a) compared to (b)



F = force magnitude due to one of the charges.

$$F_{\text{net},(a)} = \sqrt{2} F$$

(b)

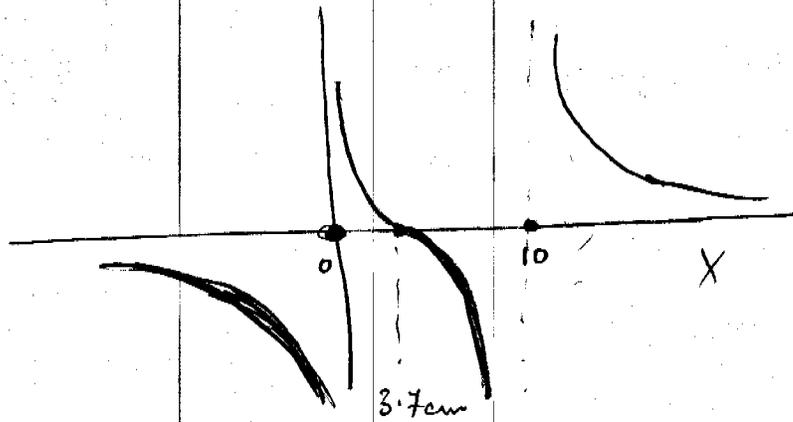


Taking components and adding

$$F_{\text{net},(b)} = \sqrt{\frac{5+2\sqrt{2}}{8}} \cdot (\sqrt{2} F) < F_{\text{net},(a)}$$

So, $a > b = c = d$

Ch 23
Pg



$$q_B = 3q_A$$

So $E = 0$ @

$$r = \frac{10}{1 + \sqrt{3}} = 3.7 \text{ cm}$$

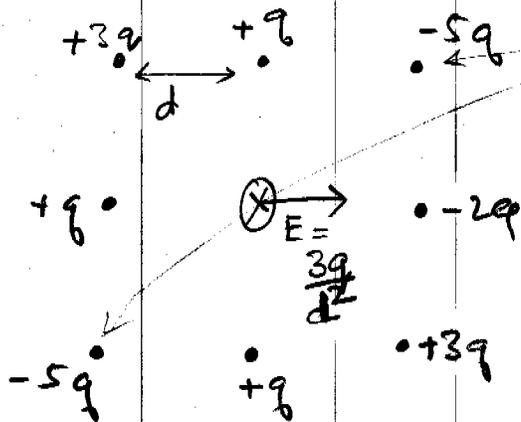
from the weaker charge (q_A)

$E = 0$ @ $x = 3.7 \text{ cm}$
for $0 < x < 3.7 \text{ cm}$, $E > 0$ ($E_{by q_A} > E_{by q_B}$)
 $x < 0$, both charges pushing to left $E < 0$
 $E \rightarrow -\infty$ as $x \rightarrow 0^-$
 $E \rightarrow 0$ as $x \rightarrow -\infty$

Also $E \rightarrow +\infty$ as $x \rightarrow 0^+$
 $E \rightarrow -\infty$ as $x \rightarrow 10^-$
 $E \rightarrow +\infty$ as $x \rightarrow 10^+$
 $E \rightarrow 0$ as $x \rightarrow +\infty$

— x —

Ch 23
P16



Field contributions due to $-5q$ at 2 ends cancel.

So also the contributions due to $+3q$ on the other diagonal

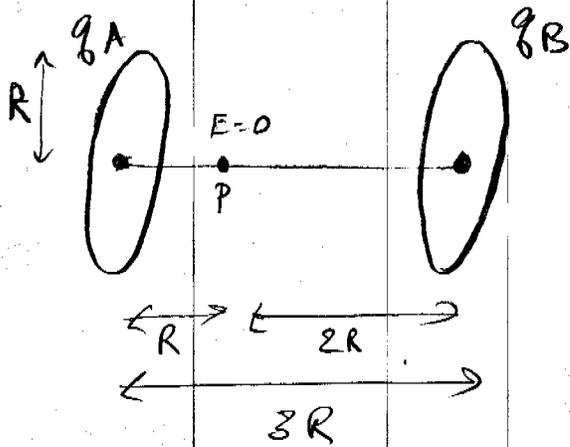
And the contributions due to $+q$ on the top & bottom sides.

Only contribution from $+q$ & $-2q$ on the sides
since those are opposite charged the
net field in the center \otimes pt is

$\frac{3q}{d^2}$ pointing to the right!

— \otimes —

Ch 23
P 22



Since P is further away from ring B, $q_B > q_A$.

[intuitively]

if they were point charges I would have said

$$q_B = 4 q_A$$

But $E_{P, \text{ by } q_A} = k q_A \frac{1}{(d^2 + R^2)^{3/2}}$ if d was distance of P from center of ring.

Here $d = R$.

$$E_{P, \text{ by } q_A} = k q_A \frac{R}{(2R^2)^{3/2}} = k q_A \frac{1}{2\sqrt{2} R^2} = \frac{1}{2\sqrt{2}} k \frac{q_A}{R^2}$$

For Ring 2

$$E_{P, \text{ by } q_B} = k q_B \frac{2R}{(5R^2)^{3/2}} = \frac{2}{5\sqrt{5}} k \frac{q_B}{R^2}$$

Since $E_P = 0 \Rightarrow \vec{E}_{P, \text{ by } q_B} = \vec{E}_{P, \text{ by } q_A}$ & $E_{P, \text{ by } q_A} = E_{P, \text{ by } q_B}$

or,

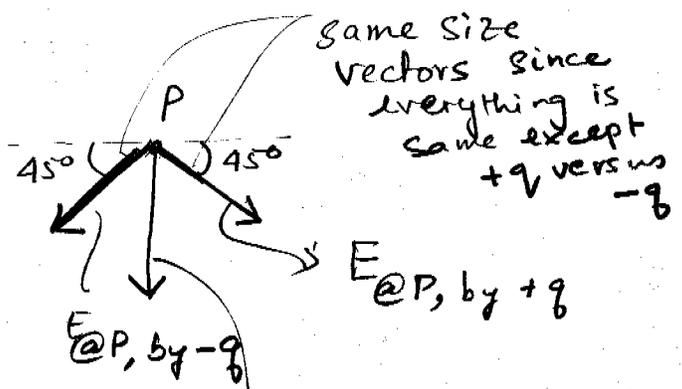
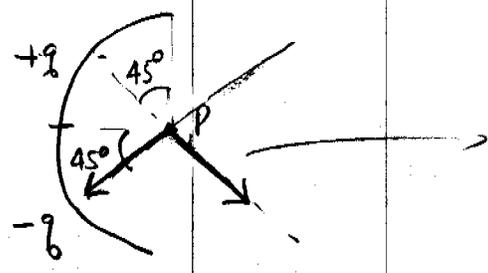
$$\frac{q_A}{2\sqrt{2}} = \frac{2 q_B}{5\sqrt{5}}$$

or

$$\frac{q_A}{q_B} = \frac{4\sqrt{2}}{5\sqrt{5}} \approx 0.5$$

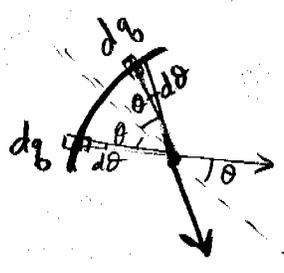
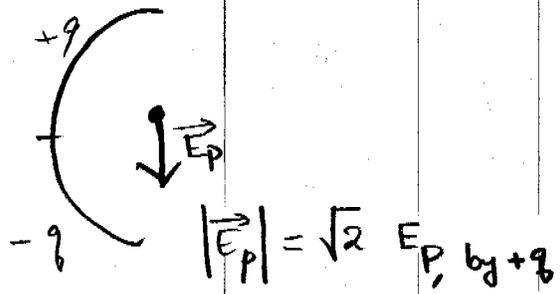
$$q_B \approx 2 q_A$$

Ch 23
P23



same size vectors since everything is same except +q versus -q

$E_p = \sqrt{2} E_{@P, by +q}$
pointing down ↓



consider 2 little elements of charge dq at an angle θ from the bisector of the $+q$ arc.
angle subtended by each little dq is $d\theta$

$$dq = \frac{+q}{\frac{\pi r}{2}} \cdot r d\theta = \frac{2q}{\pi} d\theta$$

Since the two elements would cancel out components of field perpendicular to the bisecting line, we can only count the contribution of elements, taking component along the bisector

So for these 2 elements net contribution

$$dE = 2k \frac{dq}{r^2} \cos\theta = \frac{2k}{r^2} \cdot \frac{2q}{\pi} \cos\theta d\theta$$

because of the 2 elements

taking component

Since these contributions are all along the bisector
 I can add them like scalars.

$$E_{\text{total, by } +q} = \int_{\theta=0}^{\theta=\pi/4} dE = \int_0^{\pi/4} \frac{4kq}{\pi r^2} \cos \theta \, d\theta$$

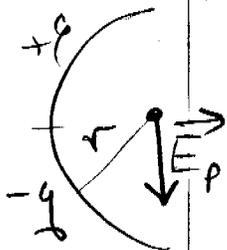
[limit = $\pi/4$
 because I have
 already accounted
 for other $\pi/4$ angle
 due to factor of 2]

$$= \frac{4kq}{\pi r^2} \left[\sin \theta \right]_0^{\pi/4}$$

$$= \frac{4kq}{\pi r^2} \sin\left(\frac{\pi}{4}\right)$$

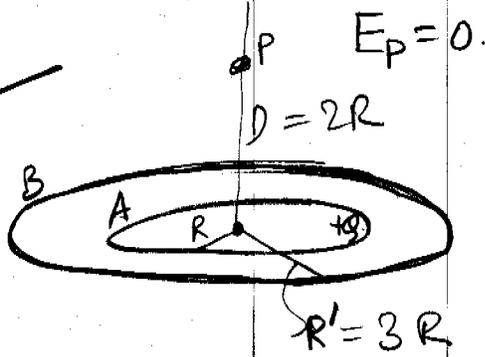
$$\Rightarrow E_P = \frac{4\sqrt{2} kq}{\pi r^2} \sin\left(\frac{\pi}{4}\right) \quad \left(\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$$

$$= \frac{4kq}{\pi r^2} = \frac{q}{\pi^2 \epsilon_0 r^2} \quad \left(k = \frac{1}{4\pi \epsilon_0}\right)$$



$$|\vec{E}_P| = \frac{4kq}{\pi r^2} = \frac{q}{\pi^2 \epsilon_0 r^2}$$

Ch 23
P 32



$\vec{E}_{P, \text{ by } A}$ (Field due to A points up)

If $E_p = 0$,
this means

$\vec{E}_{P, \text{ by } B}$ must point down

$\Rightarrow q_B$ is negative!

Small Ring, charge $+Q$: A
Large Ring, charge $= ?$: B

Now, larger the radius of the ring, further away is the charge

So for $E_{P, \text{ by } B} = E_{P, \text{ by } A}$ (magnitudes)

$q_B > Q$ (magnitude of charge)

* q_B is -ve.

$$\text{Now } E_{P, \text{ by } A} = k q_A \frac{D}{(D^2 + R^2)^{3/2}} = k +Q \frac{2R}{(5R^2)^{3/2}}$$

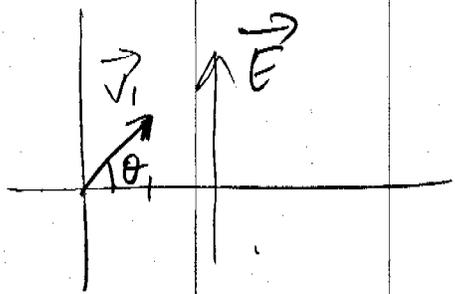
$$= \frac{2}{5\sqrt{5}} \frac{kQ}{R^2}$$

$$E_{P, \text{ by } B} = \frac{k q_B D}{(D^2 + R'^2)^{3/2}} = \frac{k q_B 2R}{(4R^2 + 9R^2)^{3/2}} = \frac{2}{13\sqrt{13}} k \frac{q_B}{R^2}$$

For fields to cancel $q_B = \frac{13\sqrt{13}}{5\sqrt{5}} Q$ and negative

or $q_B = -\frac{13\sqrt{13}}{5\sqrt{5}} Q$ (Note $|q_B| > Q$ ✓)

Ch 23
PSD

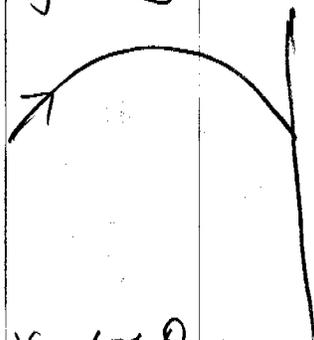


$$|\vec{v}_1| = 2 \times 10^6 \text{ m/s}$$

$$\theta_1 = 40^\circ$$

$$\vec{E} = 5 \text{ N/C } \hat{j} = E \hat{j}$$

Force on electron(e) would be ↓
So its trajectory would look like



just like a projectile.

$$v_{1x} = v_1 \cos \theta_1$$

$$v_{1y} = v_1 \sin \theta_1$$

$$\vec{v}_1 = v_{1x} \hat{i} + v_{1y} \hat{j}$$

$$\vec{F}_{\text{on } e} = -e \vec{E}$$

$$= -e E \hat{j}$$

$$[\text{Charge of electron} = -e]$$

Force will not affect v_{1x} , only v_{1y}

Mass of $e = m$ then $\vec{a} = -\frac{e}{m} E \hat{j}$

$$\therefore \vec{v}(t) = v_{1x} \hat{i} + \left(v_{1y} - \frac{e}{m} E t \right) \hat{j}$$

$$\Delta \vec{v} = -\frac{e}{m} E t \hat{j}$$

change in velocity in time t

position vector we know it hits $x = 3 \text{ m}$

$$\vec{r}(t) = v_{1x} t \hat{i} + \left(v_{1y} t - \frac{e}{m} E \frac{t^2}{2} \right) \hat{j}$$

$$\therefore \text{time } t = \frac{3.0 \text{ m}}{v_{1x}} = \frac{3}{v_1 \cos \theta}$$

∴ total velocity of electron

$$v = \sqrt{v_{ix}^2 + \left(v_{iy} - \frac{e}{m} E t\right)^2}$$

we know $t = \frac{x}{v_{ix} \cos \theta}$; $v_{ix} = v_i \cos \theta$
 $v_{iy} = v_i \sin \theta$

$$v = \sqrt{(v_i \cos \theta)^2 + \left(v_i \sin \theta - \frac{e}{m} E \frac{x}{v_i \cos \theta}\right)^2}$$

→ You can substitute values

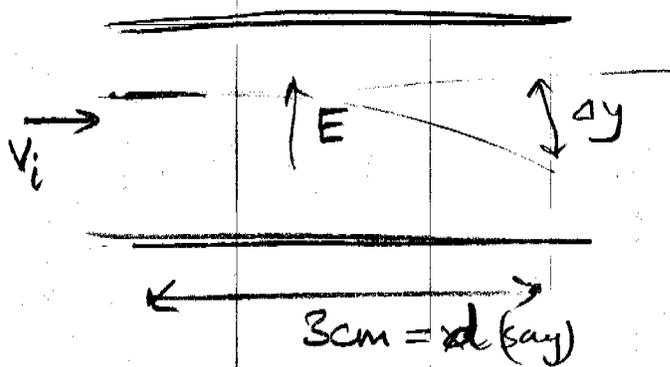
⊛ You could have avoided all the vector stuff. I wanted to show how that can be set up.

Quick method would be to argue that motion in x-dirⁿ is unchanged ⇒ time to hit = $\frac{x}{v_{ix}}$

In that time v_y would decrease by $\frac{e}{m} E \frac{x}{v_{ix}}$

$$\text{So new } v_y = v_{iy} - \frac{e}{m} E \frac{x}{v_{ix}}$$

$$\text{So now } v = \sqrt{v_{ix}^2 + v_y^2} \quad \checkmark$$



time from one end
of plate to the other
 $t = \frac{d}{v_i}$

In that time it faces
a vertically downward
constant acceleration of

$$\vec{a} = -\frac{e}{m} \vec{E} \quad \left(\begin{array}{l} \text{Force} = -eE \\ m\vec{a} = -eE \end{array} \right)$$

$$\Rightarrow \frac{\Delta \vec{v}}{\Delta t} = -\frac{e}{m} \vec{E}$$

$$\Delta \vec{v} = -\frac{e}{m} \vec{E} \Delta t = -\frac{e}{m} \vec{E} t$$

$$\therefore \text{Average vertical velocity } v_{y, \text{avg}} = \frac{|v_0| + |v_f|}{2}$$

$$= -\frac{e}{m} \frac{Et}{2} \quad \left(\begin{array}{l} \text{down} \\ \text{ward} \end{array} \right)$$

$E = |\vec{E}|$

\therefore distance covered

$$\Delta y = v_{y, \text{avg}} \cdot t$$

$$= \left(\frac{e}{m} \frac{Et}{2} \right) t = \frac{1}{2} \frac{e}{m} Et^2 = \frac{1}{2} \frac{e}{m} E \frac{d^2}{v_i^2}$$

$$\Delta y = \frac{1}{2} \frac{e}{m} E \frac{d^2}{v_i^2}$$

$$d = 30 \text{ cm}$$

$$E = 10^6 \text{ N/m}$$

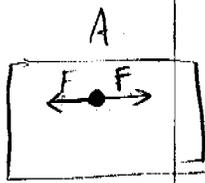
$$v_i = 3.9 \times 10^7 \text{ m/s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m = (\text{mass of electron})$$

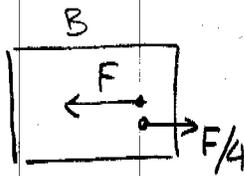
go-find it out

Ch 23
P 64

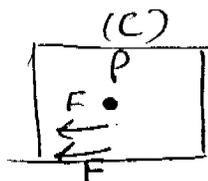


$$F_{\text{net}} = 0$$

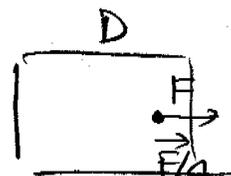
F = magnitude of force by \oplus or \ominus on \bullet P when single unit distance away



$$F_{\text{net}} = \frac{3F}{4}$$



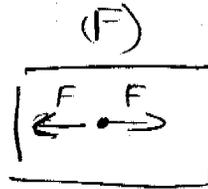
$$F_{\text{net}} = 2F$$



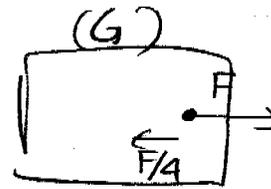
$$F_{\text{net}} = \frac{5F}{4}$$



$$F_{\text{net}} = \frac{5F}{4}$$



$$F_{\text{net}} = 0$$



$$F_{\text{net}} = \frac{3F}{4}$$

Ranking F_{net}

$$(C) > (D) = (E) > (B) = (G) > (A) = (F)$$

Ch 23
73

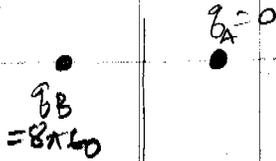
I

$$\vec{E} = \frac{k q_B}{r^2} \hat{j} = 2 \frac{N}{C} \hat{j}$$

So Ans

Direction (A)

Components (d)



II

$$q_B = -8\pi\epsilon_0$$



$$q_A = 0$$

$$\vec{E} = \frac{k |q_B|}{r^2} \hat{j} = 2 \frac{N}{C} \hat{j}$$

same as I

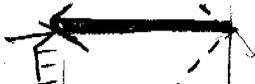
Typo should be 16πϵ_0

III

$$-16\pi\epsilon_0$$

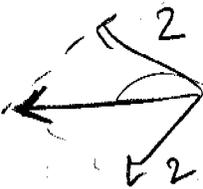
$$16\pi\epsilon_0$$

Direction (g)



$$\frac{1}{4\pi\epsilon_0} \cdot \frac{16\pi\epsilon_0}{(\sqrt{2})^2} = 2 \text{ N/C}$$

$$\vec{E} = -\sqrt{8} \hat{i}$$



$$2\sqrt{2} = \sqrt{8}$$

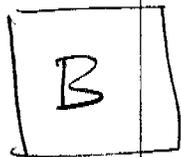
Components (g) none of the above

(a)



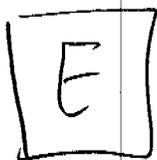
All lines that exit A also enter it again. So I am thinking that total charge in A should add up to 0.

(b)



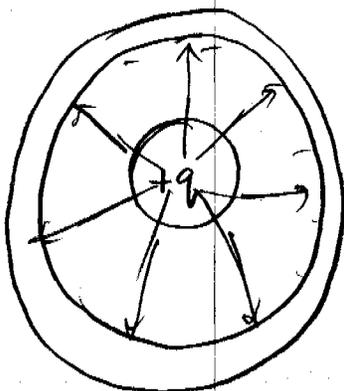
All field lines are moving out of B as if B was containing a +ve charge

(c)



C should be -ve
if $A \sim 0$
then C should be about equal
in magnitude as B

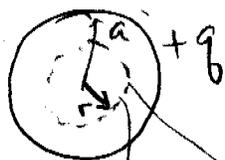
Ch 24
P 19



In region within shell
 $r < b$ field should be spherically symmetric

For $r < a$, we worked this out in class

$$E_{r < a} = \frac{q}{4\pi\epsilon_0} \frac{r}{a^3}$$



Think of a Gaussian Sphere of radius $r < a$, Then $\Phi = 4\pi r^2 E$

$$\int \vec{E} \cdot d\vec{A} = \int E dA = 4\pi r^2 E$$

$$\Phi = \frac{q_{\text{inside}}}{\epsilon_0} = q \frac{r^3}{a^3}$$

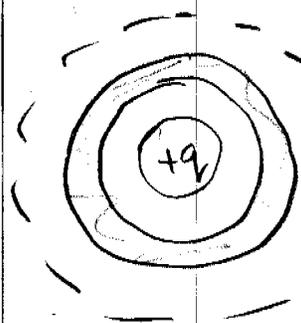
$$\Rightarrow E = \frac{q}{4\pi\epsilon_0} \frac{r}{a^3}$$

For $a < r < b$ Now $q_{\text{inside}} = +q$ (all the charge of the ball/sphere)

$$\therefore E = \frac{q}{4\pi\epsilon_0 r^2} \quad (a < r < b)$$

Within shell, $b < r < c$ ~~the area~~ we are within a metal so $E = 0$! This means that all of the $-q$ charge must be on inner surface of shell (see class slides)

For $r > c$



Spherical Gaussian Surface $r > c$

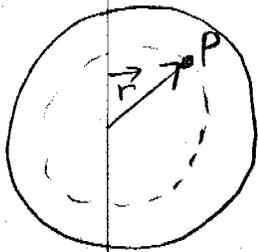
$$q_{\text{inside}} = +q - q = 0$$

$$\Rightarrow \Phi = 0$$

Since we still have spherical symmetry

E should be symmetric $\Rightarrow \underline{E = 0}$

Ch 24
P 22

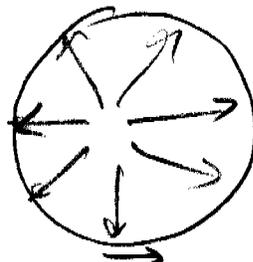


Volume charge density = ρ

~~Consider~~

(1) Field would be spherically symmetric because of the symmetry of the charge distribution

So Gaussian Surface of choice = sphere
& field would look like



pointing radially outward assuming that ρ is positive

$$\text{So } \Phi = \int \vec{E} \cdot d\vec{A}$$

$$= \int E dA = 4\pi r^2 E$$

By Gauss's law then

$$\Phi = \frac{q_{\text{inside}}}{\epsilon_0} = \frac{\rho \frac{4}{3}\pi r^3}{\epsilon_0}$$

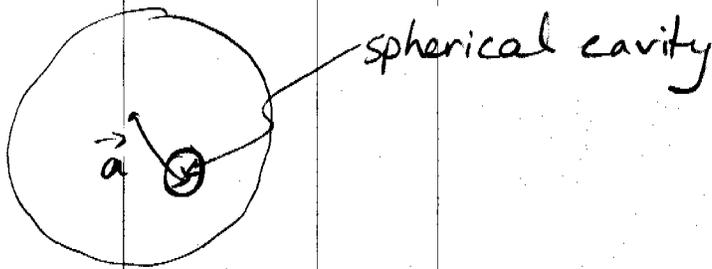
$$\text{or } 4\pi r^2 E = \frac{\rho}{\epsilon_0} \frac{4}{3}\pi r^3$$

$$\text{or } E = \frac{\rho r}{3\epsilon_0} \Rightarrow \vec{E} = \frac{\rho r}{3\epsilon_0} \hat{r} = \frac{\rho \vec{r}}{3\epsilon_0}$$

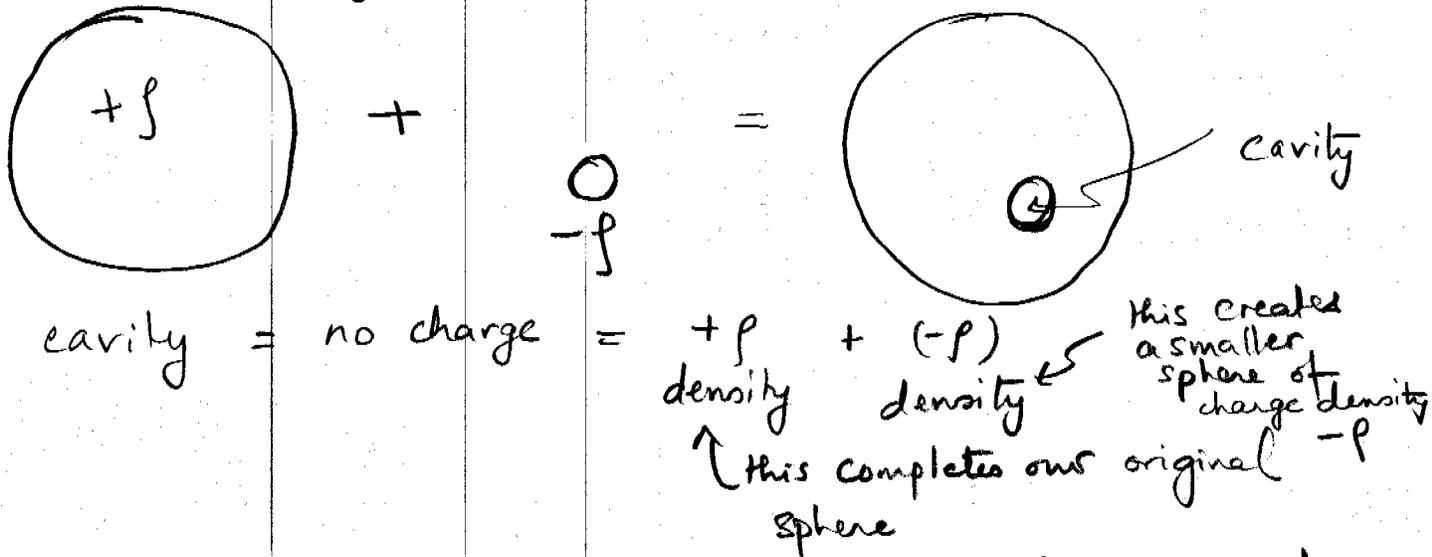
$$\boxed{\vec{E} = \frac{\rho \vec{r}}{3\epsilon_0}}$$

$\vec{E} = E \hat{r}$
 $d\vec{A} = dA \hat{r}$
 \hat{r} is radial vector
 $\hat{r} = \frac{\vec{r}}{r}$

Ch 24
P 22 (b)

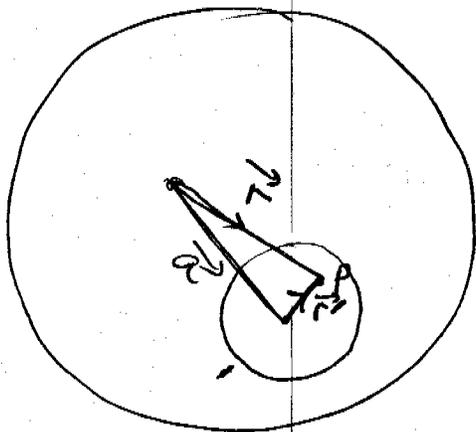


Model sphere with cavity as superposition of 2 oppositely charged spheres



So Field inside cavity = Superposition of field due to large sphere (positive ρ) \curvearrowright A
 small sphere (negative ρ) \curvearrowright B

Consider a point P inside cavity



~~Field due~~ \vec{E}_p due to A = $\frac{\rho \vec{r}}{3\epsilon_0}$

\vec{E}_p due to B = $-\frac{\rho \vec{r}'}{\epsilon_0}$

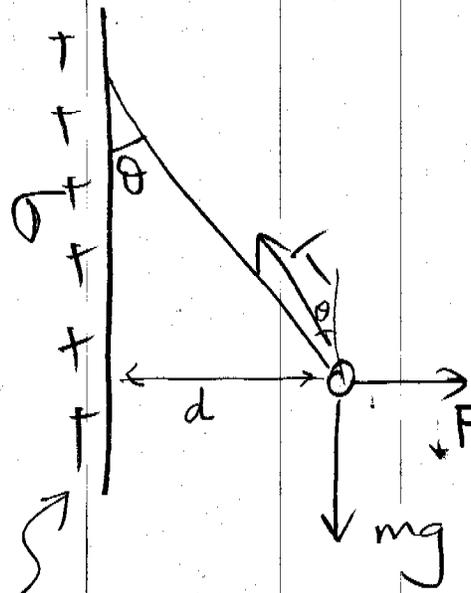
$\vec{E}_{p, \text{total}}$ = $\frac{\rho \vec{r}}{3\epsilon_0} - \frac{\rho \vec{r}'}{\epsilon_0}$

\vec{E}_p = $\frac{\rho}{3\epsilon_0} (\vec{r} - \vec{r}')$

\vec{E}_p = $\frac{\rho \vec{a}}{3\epsilon_0}$

or,
or

Ch 24
P 85



For ball to be in equilibrium

$F_{net} = 0$

Assuming that sheet is very very large. It acts as a infinite sheet

$E =$ field due to infinite sheet

$E = \frac{\sigma}{2\epsilon_0}$

[See HW 9 P#2 towards the end I mention this]

For equilibrium

$$\tan \theta = \frac{F_e}{mg} = \frac{q\sigma}{2\epsilon_0 mg}$$

$$\text{or } \sigma = \frac{2\epsilon_0 mg \tan \theta}{q} \quad \checkmark$$

We know all quantifies

P35

continued

Now note, if m was large, we would need a larger charge ^{density} in the sheet to hold it the same distance away

So if $m \uparrow$ $\sigma \uparrow \rightarrow$ bingo \rightarrow matches with the math
(note m is in the numerator of expression)
should

If θ is larger, everything else being the same, that means that the F_e is larger so that the ball is pushed away further. This implies a larger σ .

\rightarrow Mathematically if $\theta \uparrow \tan \theta \Rightarrow \sigma \uparrow$ in our expression

awesome! matches again

finally, if the ball was at the same angle for a larger q , that means that the sheet must have a weaker σ , so that F_e remains the

same \Rightarrow if $q \uparrow \sigma \downarrow \rightarrow$ & note q is in

the denominator of our expression so everything fits!

Ch 24
P 49

(a) Different parts of the box are at different distances from the center. So the field at different points on a face of a cube is different \Rightarrow density of field lines would be different

most dense at center. & spreads out towards the corners

(b) \hookrightarrow # lines through A < # lines through B

Since $E_A < E_B$ (since $r_A < r_B$)

$$\Rightarrow \Phi_A < \Phi_B$$

\hookrightarrow Flux corresponds to # lines crossing surface

(c)

Ch 24

51

(a)

- (i) True
- (ii) False
- (iii) False
- (iv) False
- (v) True
- (vi) false

(b)

- (i) False
- (ii) True
- (iii) false
- (iv) false
- (v) false
- (vi) True

Gauss's law is useful if field is symmetric but it is still valid if the field is not symmetric!