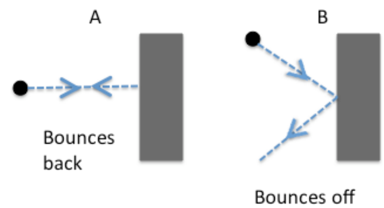
Multiple Choice (5 points each; No partial credit)

In each case, write only the answer choice in the box on the right

(1) A ball of mass = 10g slides on a frictionless table and strikes a barrier in two different cases as shown in the figure. In A, it bounces right back to retrace its path; in B, it bounces diagonally (path shown in dotted line). The initial speed of the ball is the same in both cases and no energy is lost in either collision. Which of the following is true?



(A) There is not enough information to determine which case has a greater change in momentum

(B) The change in the ball's momentum is zero for both cases The direction of the ball's motion is changing which means that the velocity vector is changing and so the change in the ball's momentum in each case is non-zero

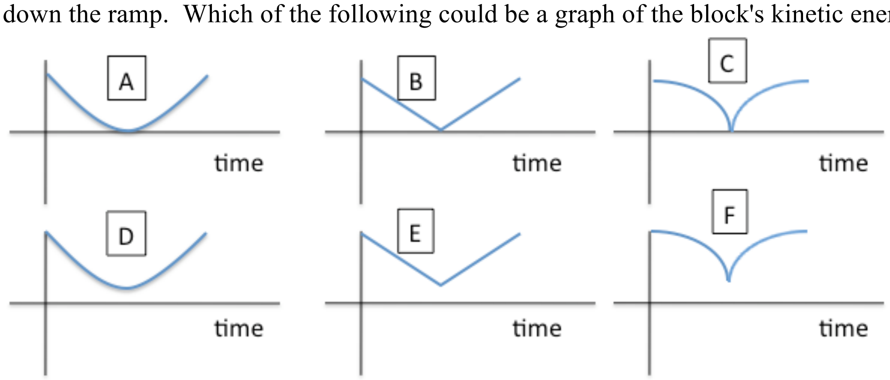
$\boxed{C}$

(C) The change in the ball's momentum is greater for case A than for B: This is correct! CaseA: $\mathbf{\Delta p} = (mv)\mathbf{i} - (-mv)\mathbf{i} = (2mv)\mathbf{i}$. Case B: $\mathbf{\Delta p} = (2mv\cos(\theta))\mathbf{i}$, where θ is the angle between the initial velocity and the horizontal line.

(D) The change in the ball's momentum is the same in both cases (but not zero)

(E) The change in the ball's momentum is greater for case B than for A

(2) A block slides up a ramp with an initial speed of v_0 up the ramp, reaches the top, and then slides back down the ramp. Which of the following could be a graph of the block's kinetic energy (versus time)?



$\boxed{A}$

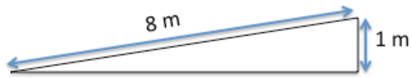
KE first decreases as the box goes up the ramp, is zero right at the top of the motion, and then increases as the box slides down. But the rate of change of KE is not linear with time (it is linear with height), it decreases faster initially and then slower at the top. So, the answer is A.

(3) Two identical balls (of equal mass) are thrown vertically upwards. Ball 2 goes twice as high as Ball 1. If the impulse given to ball 1 is I_1 and the impulse given to ball 2 is I_2, then I_2/I_1 = ?

(A) 2	(D) 1/4	(G) $1/\sqrt{2}$
(B) 1/2	(E) 1	
(C) 4	(F) $\sqrt{2}$	

$\boxed{F}$

Ball 2 goes higher, so it must have gotten a larger impulse. I_2/I_1 is greater than 1. Now, Ball 2 goes twice as high, so at the top, its potential energy is twice that of ball 1. So, at the bottom its $KE_2 = 2KE_1$. Since impulse = change in (mv) and K.E. = $1/2(mv^2)$ so, $I_2/I_1 = \sqrt{2}$. (In another version I asked for I_1/I_2 = so that will be $1/\sqrt{2}$. In another version of the exam, it is Ball 1 that goes twice as high -- in that case $I_1/I_2 = \sqrt{2}$)



(4) To lift a 10 kg box 1 m off the ground, you could put it on very good rollers and push it up an 8 m ramp, applying a minimum force up the ramp of about

(A) 75 *N*
(B) 50 *N*
(C) 12.5 *N*
(D) 25 *N*
(E) 100 *N*

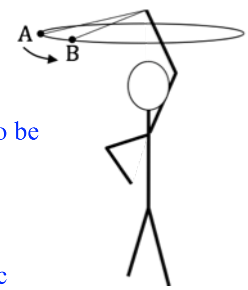
$\boxed{C}$

Well, the total PE gained by the box = (10kg)(10m/s^2)(1m) = 100 J. But if I take it up the ramp, then work done is FΔx = F(8m) and so F = 100/8 N = 12.5 N

Short Answer (15 points each)

No credit without explanation; you can continue to write on the back of the sheet if needed

(5) I am swinging a ball around my head in a horizontal circle as shown. The ball has a constant speed during this motion. The speed of the ball is sufficiently small that the air-resistance can be neglected. **For each of the questions below, briefly explain your reasoning. Also, make sure that your answers are consistent with each other.**



(a) Is the momentum of the ball conserved as the ball moves?

The momentum of the ball is not conserved as the ball moves. Since the ball is moving in a circle, its velocity vector is changing in direction from one instant to another. Since **p**=m**v**, the momentum must also be changing as the velocity vector changes.

(b) Is the total mechanical energy (potential + kinetic energies) of the ball conserved as the ball moves?

The total mechanical energy of the ball is conserved. The speed of the ball does not change. So, its kinetic energy is constant throughout its motion. Also, since the ball is moving in a horizontal circle, its height is constant and so its gravitational potential energy is constant throughout its motion. Putting these two together, the sum of its kinetic and potential energies (i.e., its total mechanical energy) must be conserved throughout its motion.

(c) Is there a net work done on the ball in moving from point A to point B along its path?

No net work is done on the ball in moving from point A to point B. There are two forces on the ball -- Tension from the string and the gravitational attraction by the Earth. At all points on the ball's trajectory, both of these forces are perpendicular to the motion of the ball. So, Work done = $\mathbf{F}\bullet\Delta\mathbf{x} = |\mathbf{F}||\Delta\mathbf{x}|\cos(\theta) = 0$ because here $\theta = 90°$. And this is consistent with the total mechanical energy being conserved.

(d) Is there a net impulse on the ball in moving from point A to point B along its path?

Yes there is. Since the ball is going around in a circle there is a net force on the ball - directed towards the center of the circle and having magnitude mv^2/r where, m is the mass of the ball, v is its speed, and r is the radius of the circle. So there is a net impulse on the ball equal to $\mathbf{F}_{avg}\Delta t$ (Δt is the time taken by the ball to move from A to B; and $|\mathbf{F}_{avg}| = mv^2/r$ and direction of force is radially inwards from the point midway between A and B). And this is consistent with the momentum of the ball not being conserved. Since there is a net impulse on the ball, its momentum has to change.

(e) For **one of above questions** give a ***good*** explanation for why someone might think otherwise.

Someone might think that the momentum of the ball is conserved - because the speed of the ball does not change and so the quantity mv does not change during its motion.

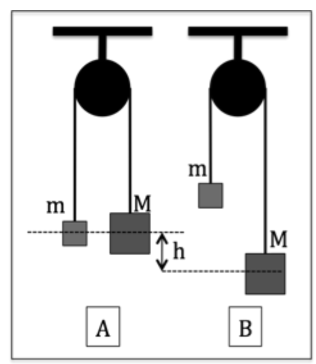
Someone could reasonably think that there is net work done on the ball. Since the ball is moving in a circle, there has to be a net centripetal force on it - and that multiplied by the displacement of the ball could provide the work done on the ball.

(f) Briefly explain what is wrong with the reasoning in (e)

For one, it is true that the quantity mv is constant throughout the ball's motion - but that just means that the magnitude of the ball's momentum does not change. What that student is ignoring is that momentum is a vector and to decide if it is conserved or not, we need to pay attention to both the magnitude and the direction of momentum. In case of the ball here, it is the direction of momentum that is changing - and so the momentum is not conserved even though its magnitude remains the same always.

For the second one, it is true that there is a net centripetal force on the ball. But that force does not do any work on the ball because the centripetal force is always perpendicular to the direction of movement. For work to be done, it is not sufficient that there be a net force, but there needs to be a net force *in the direction* of displacement! This is because work is not simply a product of net force and displacement but a *dot product* of the force and displacement vectors - and the dot product also depends on the cosine of the angle between the force and displacement vectors. In this case, the net force is centripetal (towards the center) and thus has no tangential component along the ball's displacement at any point on its trajectory resulting in net work of 0.

(6) Two blocks, of masses M and m **(M>m),** hang by a frictionless rope (negligible mass) from a frictionless pulley (negligible mass) as shown. The blocks start from rest in the position shown in A.



(a) Using conservation of energy, find out the velocity of the block M when it has moved down by a distance *h* (as shown in figure B) in terms of M, m, *h*, and g. ***Don't just write equations!*** **Explain the reasoning behind any equations you construct.**

Considering the two blocks as one system, the only external force on the blocks is the gravitational force, and there is no significant friction anywhere. So, we can say that the total mechanical energy of the blocks is conserved. Initially, the blocks are at rest in A and then they are moving in B, so that they gain kinetic energy in going from A to B. From the conservation idea above, the gain in kinetic energies implies that the blocks as a whole lost potential energy and that $\Delta KE + \Delta PE = 0$.

Now since the blocks are tied to the rope, they must be traveling at the same speed at all times (before M hits the ground). So if v is the speed of M at position B, then v must be the speed of m at position B. In other words, the gain in KE of the two blocks in going from A to B is: $\Delta KE = \frac{1}{2} mv^2 + \frac{1}{2} Mv^2$.

Again in going from A to B, M loses Mgh amount of PE. Since the two blocks always have the same speed (or even simply, since the rope remains the same length), if block M goes down by a distance h then m must have risen by a distance h. So, block m gains mgh amount of PE. The change in the PE of the two blocks is thus:

$\Delta PE = -Mgh+mgh = -(M-m)gh$ {It makes sense that ΔPE is negative: I argued that the two block system loses PE and the negative change confirms that}

The conservation of energy them gives us.

$\Delta KE+\Delta PE=0$,

$\frac{1}{2} mv^2 + \frac{1}{2} Mv^2 -(M-m)gh = 0$ or, $v = \sqrt{2\frac{M-m}{M+m}gh}$

(c) Check to see if the expression for velocity that you get above makes sense: Show how the mathematical dependence of the velocity of M on the other variables, M, m, and *h* is supported by your sense of the physical situation.

With respect to h, the answer is simplest. Mathematically, the expression for v shows that larger h would imply a larger v. Common sense would agree. We know that the blocks are accelerating ... and so the lower M goes, the higher its velocity. Which is what the math is saying.

For m, well, if we had a larger m, we can imagine it holding back more on M (or you can think that it would be more difficult for M to pull m up), i.e., the acceleration of the blocks would be lesser which would argue for a smaller v in position B. In terms of energy we can argue that more of PE is needed to raise m and a smaller portion of the PE loss of M can go in increasing the KE of the blocks, resulting in a smaller v. Mathematically, if we look at the expression (M-m)/(M+m), we see that as m increases, the numerator decreases but the denominator increases, implying that the size of that ratio would decrease. This means that v would be lesser for a larger m -- the implications of the expression we got for v matches well with the common sense argument above.

For M, now, at M=m, the ratio (M-m)/(M+m) is 0, which makes sense because if the masses are equal, they will be balanced and won't move at all (v=0). If M is very, very large compared to m, then the ratio approaches 1. So as M increases, we would expect the ratio to increase in size [you can also see that be rewriting $\frac{M-m}{M+m}=1-2\frac{m}{M+m}$]. Thus, the expression I got for v implies that v increases if M increases (other quantities being kept the same). Now thinking about the physical system, if M increases, it would be easier for it to pull up m resulting in a larger acceleration (as a system the force on the blocks as (M-m)g and that is increasing) and a larger velocity at position B. So, the physical reasoning supports the mathematical equation that we got for v.

So the expression for v that I got makes sense: the implications of the math are aligned with how we would expect the system to behave. This "checking" does not *guarantee* that the expression I got is perfectly correct, but it provides a larger confidence in my math if the math aligns with physical intuition. That combined with a thorough check of the derivation makes me pretty sure that I have not made mistakes in my calculations.